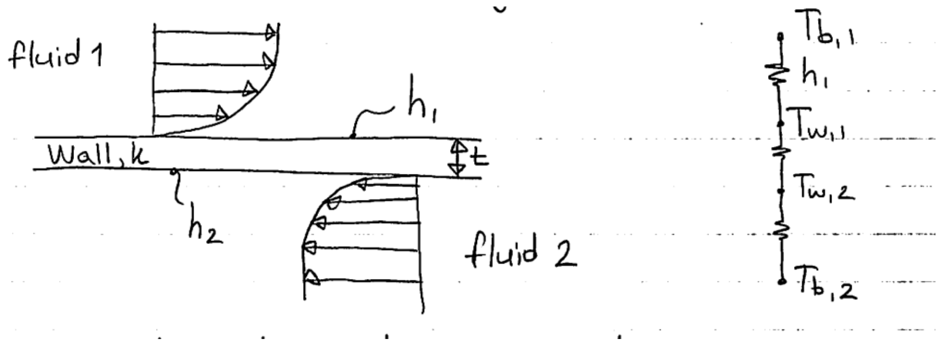
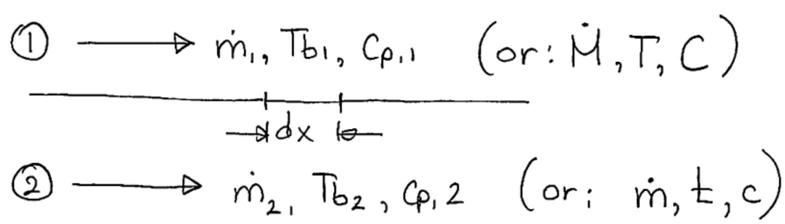
Heat Exchanger Problem Calculation (LMTD and Effectiveness-NTU method)

(Reorganization of the Lecture Notes from Professor Nenad Miljkovic)



From our resistance network, we can write $\mathrm{U}=1/(\frac{1}{\mathrm{h}_1}+\frac{\mathrm{t}}{\mathrm{k}}+\frac{1}{\mathrm{h}_2})$

Now we can analyze out heat exchanger: (Perimeter P)



Across the element dx:

$$\mathrm{dq}=-\dot{M}CdT=\dot{m}cdt$$

$\mathrm{dT}=-\mathrm{dq}/\dot{M}C$ and $\mathrm{dt}=\mathrm{dq}/\dot{m}c$

$$\mathrm{d(T-t)}=\frac{\mathrm{dq}}{\dot{M}C}*\{-1-\frac{\dot{M}C}{\dot{m}c}\}$$

But we know that $\mathrm{dq}=\mathrm{UPdx(T-t)}$, where $\mathrm{Pdx}=\mathrm{dA}$.

$$\frac{\mathrm{d(T-t)}}{\mathrm{T-t}}=\frac{\mathrm{U}}{\dot{M}C}*\left\{-1-\frac{\dot{M}C}{\dot{m}c}\right\}dA$$

Integrating both sides from inlet (a) to outlet (b), we obtain

$$\ln\{(\mathrm{T_b}-\mathrm{t_b})/(\mathrm{T_a}-\mathrm{t_a})\}=\mathrm{UA}/\dot{M}C*\left\{-1-\frac{\dot{M}C}{\dot{m}c}\right\}$$

For the entire heat exchanger, we know:

$$-\dot{M}C(T_b-T_a)=\dot{m}c(t_b-t_a)$$

Back substitute:

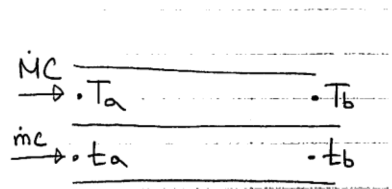
$$\ln\{(\mathrm{T_b}-\mathrm{t_b})/(\mathrm{T_a}-\mathrm{t_a})\} = \mathrm{UA}/\dot{M}C(T_b-T_a) * \{(T_a-T_b)+t_b-t_a\}$$

$$-\mathrm{q} = \dot{M}C(T_b-T_a) = UA\{(T_a-T_b)+t_b-t_a\}/\ln\{(\mathrm{T_b}-\mathrm{t_b})/(\mathrm{T_a}-\mathrm{t_a})\}$$

Since $\mathrm{q} = \mathrm{UA\Delta T}$

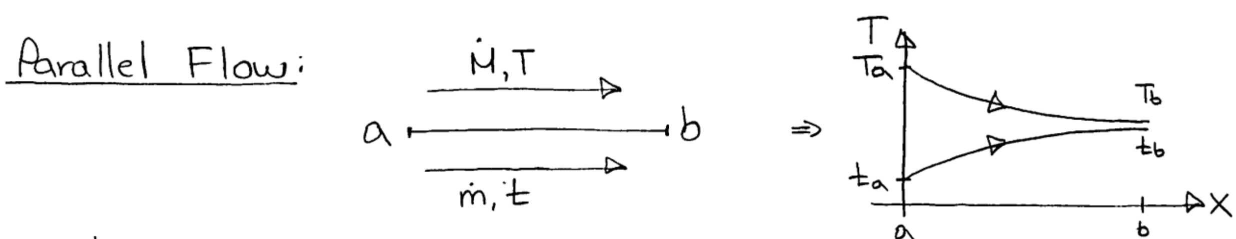
Now we can write $\Delta\mathrm{T} = \mathrm{LMTD}$ (Log Mean Temperature Difference)

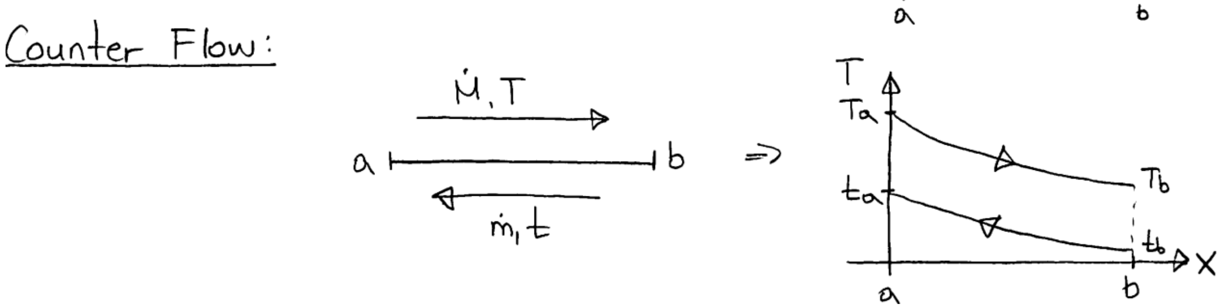
$$\Delta\mathrm{T} = \{(T_b-t_b)-(T_a-t_a)\}/\ln\{(\mathrm{T_b}-\mathrm{t_b})/(\mathrm{T_a}-\mathrm{t_a})\}$$



Note that it doesn't matter which side is a and b. You can reverse it and still get the same answer. (i.e. switch a and b)

It also doesn't matter which direction the flows are traveling in, as long as "a" and "b" refer to same physical end of the heat exchanger.





Think of the LMTD as a convenient way to define a $\Delta\mathrm{T}$ between two streams whose temperature is continuously varying.

Special case: Balanced counter flow heat exchanger

Assuming $\dot{M} = \dot{m}$ and C=c, then the LMTD becomes undefined as $\Delta\mathrm{T_b} = \Delta\mathrm{T_a}$ and LMTD=$\frac{0}{0}$.

The way to resolve this is the following:

Suppose $\Delta T_b = \Delta T_a + \varepsilon$, where $\varepsilon \ll 1$

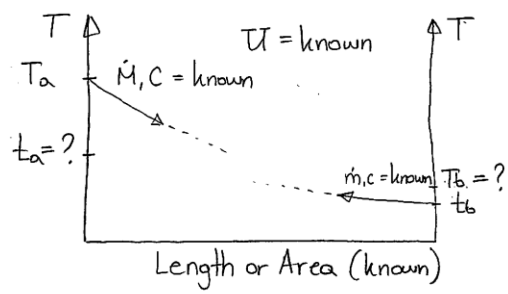
$$\text{LMTD} = \frac{\Delta T_a + \varepsilon - \Delta T_a}{\ln\left\{\frac{\Delta T_a + \varepsilon}{\Delta T_a}\right\}} = \varepsilon / \ln\left\{1 + \frac{\varepsilon}{\Delta T_a}\right\} \approx \Delta T_a = \Delta T_b = \Delta T$$

In general, we can say the following:

Parallel Flow	Counter Flow
Disadvantage 1: Large temperature difference at one end of the heat exchanger causes added thermal stresses and early failure.	Advantage 1: More uniform ΔT minimizes the thermal stresses throughout the heat exchanger.
Disadvantage 2: The outlet temperature of the cold fluid never exceeds the outlet temperature of the hot fluid. Less efficient.	Advantage 2: The outlet temperature of the cold flow can approach the highest temperature of the hot fluid. More efficient.
	Advantage 3: More uniform ΔT produces a more uniform q.

$\varepsilon - NTU$ method (Effectiveness-NTU method)

Note that in most heat exchanger design problems, we don't know the fluid outlet temperatures in advance.



Now we can define a quantity called the capacity

$C_h = (\dot{M}C)_{hot} [\frac{W}{K}]$; $C_c = (\dot{m}c)_{cold} [\frac{W}{K}]$

To solve, we could guess exit temperature, solve for $Q_h = Q_c = C_h \Delta T_h = C_c \Delta T_c$

Then we would have to calculate Q from UALMTD and check against our previous answer. If differing, we would guess another exit temperature and try again.

However, we can make our lives much easier with

$$\varepsilon = \frac{\text{actual heat transferred}}{\text{maximum heat that could possibly be transferred from one stream to other}}$$

Mathematically, this is equal to:

$$\varepsilon = \frac{C_h(T_{h,in} - T_{h,out})}{C_{min}(T_{h,in} - T_{c,in})} = \frac{C_c(T_{c,out} - T_{c,in})}{C_{min}(T_{h,in} - T_{c,in})}$$

where C_{min} is the smaller of $C_h \& C_c$.

So we write: $Q = \varepsilon C_{min}(T_{h,in} - T_{c,in})$

We can also define: $NTU = \frac{UA}{C_{min}} = \frac{heat\ rate\ capacity\ of\ heat\ exchanger}{heat\ capacity\ rate\ of\ flow}$ (dimensionless)

Using energy balances and simplification, we can solve for our two cases:

i. Parallel flow:

$$-\left(\frac{C_{min}}{C_c} + \frac{C_{min}}{C_h}\right) NTU = \ln[-\frac{\left(1 + \frac{C_c}{C_h}\right)\varepsilon C_{min}}{C_c} + 1]$$

Solving for ε, we obtan:

$$\varepsilon = \frac{1 - \exp\left[-\left(1 + \frac{C_{min}}{C_{max}}\right)NTU\right]}{1 + \frac{C_{min}}{C_{max}}} = f(\frac{C_{min}}{C_{max}}, NTU)\ \text{only}$$

ii. Counter flow:

$$\varepsilon = \frac{1 - \exp\left[-\left(1 - \frac{C_{min}}{C_{max}}\right)NTU\right]}{1 - \frac{C_{min}}{C_{max}}\exp\left[-\left(1 - \frac{C_{min}}{C_{max}}\right)NTU\right]} = f(\frac{C_{min}}{C_{max}}, NTU)\ \text{only}$$

We can plot our results in a more useful form

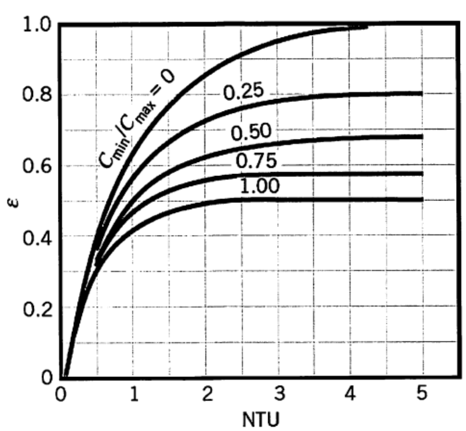


FIGURE 11.10 Effectiveness of a parallel-flow heat exchanger (Equation 11.28).

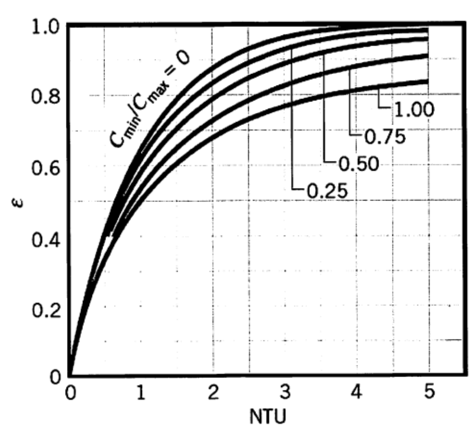


FIGURE 11.11 Effectiveness of a counterflow heat exchanger (Equation 11.29).